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LETTER TO THE EDITOR

Infinite conductivity in general relativistic magnetohydrodynamics

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Abstract. It is pointed out that in general relativistic magnetohydrodynamics Ohm's law does not determine the current density J^a in the limit of infinite conductivity, but that it is determined by the electromagnetic field equation as in nonrelativistic magnetohydrodynamics. It is also shown that the relation $2\omega.H = \epsilon$ derived by Yodzis assuming $J^a = \epsilon u^a$ for infinite conductivity, can also be obtained by a modified argument if the above view is taken.

In an interesting paper on relativistic magnetohydrodynamics, Yodzis (1971) has shown, among other things, that the proper charge density ϵ of a conducting fluid is related, in the limit of infinite conductivity, to the magnetic field H^a and the local angular velocity ω^a of the fluid by

$$\epsilon = 2\omega.H. \tag{1}$$

In the derivation of this result, Yodzis assumed that the relativistic version of Ohm's law

$$J^a = \epsilon u^a + \sigma E^a \tag{2}$$

(Yodzis's notation will be used throughout this letter) is, in the limit of infinite conductivity,

$$J^a = \epsilon u^a. \tag{3}$$

This assumption does not correspond to the limiting procedure in nonrelativistic magnetogydrodynamics (Hughes and Young 1966 or Roberts 1967). All that can be said as σ tends to infinity is that E^a must tend to zero in order to keep $J^a - \epsilon u^a$ finite, but not necessarily zero. As in the nonrelativistic case, Ohm's law does not determine J^a in this limit. It is determined by the electromagnetic field equations.

The purpose of this communication is to point out that equation (1) can still be established assuming only that E^a tends to zero as σ tends to infinity, rather than Yodzis's more restricting assumption. Contrast Yodzis's equation (9)

$$u^a{}_{|b}D^b + u^a D^b{}_{|b} - u^b{}_{|b}D^a - u^b D^a{}_{|b} - \eta^{abcd}(u_{c|b}H_d + u_c H_{d|b}) = \epsilon u^a + \sigma E^a \tag{4}$$

with u_a. This equation is valid for finite conductivity. Using obvious identities this contraction gives

$$D^b{}_{|b} - \alpha_a D^a + 2\omega.H = \epsilon \tag{5}$$

where $\alpha_a = u_{a|b}\, u^b$ is the acceleration of the fluid element. The term σE^a drops out in the contraction. Now take the limit of infinite conductivity. As $D^a = \lambda E^a$ and $E^a = 0$ in this limit, (5) simplifies to (1) as claimed. Equation (5) is of interest in its own right.

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References

Yodzis P 1971 *Phys. Rev.* D 3 2941–5
Hughes W F and Young F J 1966 *The Electrodynamics of Fluids* (New York: Wiley) p 155
Roberts P H 1967 *An Introduction to Magnetohydrodynamics* (London: Longmans) p 36